

# Optimal planning of order-based operations for intermodal freight transportation

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**Abstract**—In this paper, a planning problem is analysed for a consolidation transportation system in order to serve different transportation requests (i.e., orders) with different origins and destinations and different characteristics. The considered transportation system is a railway network provided with innovative terminals (where handling operations are realized very much faster than in traditional terminals). Therefore, a box can in general be loaded on different trains before reaching its destination.

This planning problem is faced at two decisional levels, corresponding to the off-line planning and the on-line monitoring and control of the system. The off-line planning procedure aims at defining a detailed transportation plan in order to satisfy all requests and minimize the overall cost. This is achieved by a preprocessing analysis and, then, the solution of a mathematical programming problem. The on-line planning phase aims at facing a criticality happening during the system operation. At this purpose, a three-step procedure is devised in which the impact of the criticality is evaluated and, then, a local solution is searched; if it is not found, a global optimization procedure is run.

## I. INTRODUCTION

Intermodal transportation can be defined as the transportation of a person or a load from one origin to one destination by adopting different transportation modes such that the change between different modes is realized in intermodal terminals. A significant part of freight trade is moved in containers. Container traffic has continuously increased in the last years and it is actually increasing, because of many advantages, such as the standardization, greater safety levels and reduced cargo handling operations. In the field of intermodal transportation, an important distinction is that between consolidation transportation (where a convoy moves freight for different customers and often with different origins and destinations, as in the present paper) and customized transportation (in which a transportation service is reserved to a particular customer).

Some general review works in the field of planning and optimization of logistic and freight transportation systems are [1], [2], [3], [4], [5]. This paper is devoted to planning transportation operations in railway networks in order to satisfy a transportation demand represented as a set of orders (indicating an origin, a destination, the quantity of containerized goods to be moved, time delivery specifications, and so on). Many research works can be found about planning railway operations, in which the main objective is the optimal scheduling and routing of trains (see [6] for a review work on

this field). For instance, in [7] a multiobjective programming model is defined in order to plan passenger train services; in [8] a model is proposed for a scheduling recurring rail deliveries of bulk commodities from a set of suppliers to a set of customers. Instead, the present work does not aim at planning train schedules, while the objective is the definition of optimal loading/unloading plans, i.e. the optimal assignment of boxes to trains. A peculiar characteristic of this planning procedure is that boxes of the same order are not forced to follow the same path on the network. This is a specific problem for which no similar research papers can be found in the literature.

The main focus of this paper is on defining a planning procedure for order-based transportation operations in a railway network, both in *off-line* and *on-line* frameworks. As a matter of fact, as it usually happens when managing complex systems, two decisional phases must be addressed, corresponding to an off-line static planning procedure and to an on-line planning scheme able to react to system criticalities. More in particular, we suppose that all orders are known at a given time instant in which an off-line planning is realized (as already described in [9]). Then, when the system is working, it can happen that some data used to make the off-line planning are no more valid because of some critical events (for instance, a train is late and the considered timetable is no more correct). In this case, an on-line planning procedure is applied, as described in the present paper. Of course, the on-line planning procedure must be consistent with the off-line one, sharing the same objective function and considering the same constraints.

The paper is organized as follows. In Section II the considered problem is analysed and the objective of the paper is highlighted. Sections III and IV are, respectively, devoted to the detailed description of the off-line and on-line planning procedure. In order to better clarify these planning approaches, an example is provided in Section V. Finally, some conclusive remarks are reported in Section VI.

## II. PROBLEM DESCRIPTION

The problem described in this paper is based on a new concept of freight railway network where railway terminals are provided with innovative transfer systems for containers and swap bodies. Actually, in the last years, new technologies have been designed and developed in order to make handling

operations faster, yielding a strong improvement in intermodal operations. In traditional terminals, loading and unloading movements are realized by means of cranes which handle containers vertically, preventing from loading/unloading a train under the electric line. Then, trains need to be moved out of the rail line by diesel locomotives, causing very long delays at terminals. For this reason, traditional rail freight transport is realized with point-to-point trains without intermediate stops, which result profitable only for large volumes of goods. On the contrary, when innovative systems (based on a horizontal transfer) are applied in the terminals of a network, handling operations are certainly faster than the traditional ones. In these cases a new concept of railway transportation is developed: a container can move from an origin to a destination node by changing different trains on the path, exactly as it happens to passengers.

These innovative railway networks need to be efficiently managed and organized and this gives rise to new planning problems related to transportation operations on such networks and, more in general, on intermodal networks (composed of road pickup and delivery routes and rail routes). It is to be noted that rail operations and road operations represent two separate phases in the overall transportation network. Rail is generally adopted for longer transportation operations associated with high volumes of goods whereas road is dedicated to shorter distances and smaller volumes. Moreover, these two phases involve different transportation resources that are, in some sense, “synchronized” within terminals. When dealing with planning transportation operations, it is thus possible and even more effective to separate decisional aspects relevant to rail operations from what concerns road operations. Moreover, road operations are more flexible than the rail ones; for this reason the planning problem on railway networks must be faced firstly, assuming to have infinite road resources present at terminals whenever necessary, and this is just the purpose of the present paper.

Specifically, this paper deals with the definition of an optimization procedure for planning transportation operations in order to satisfy a transportation demand represented as a set of orders. An order corresponds to a request of moving a certain quantity of products from an origin to a destination by using the trains available in the network: each order is then defined by specifying the quantity and type of boxes (either containers or swap bodies) to be transported, the origin and destination places and the requirements on delivery times. Since we only consider the railway network, without deciding about road transportation, we suppose that goods are provided at an origin railway terminal and they are collected at a destination railway terminal. As already pointed out, an important aspect of this planning procedure is that boxes belonging to the same order can be split over the network, hence covering different routes and being loaded on different trains, in order to optimally use the transportation services considering that trains and terminals are limited resources.

As it usually happens when complex systems must be managed and planned, two decision phases are considered:

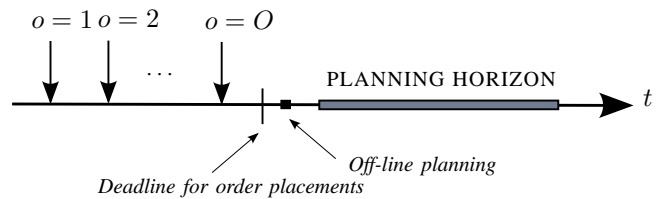


Fig. 1. On-line and off-line planning phases.

the off-line planning phase and the on-line control phase. The former phase is devoted to the definition of the optimal way of serving orders with the available railway network; it is based on static and forecasted data. The result of this phase is a detailed plan that will be applied to the system on-line. Though, the system operation is generally influenced by stochastic aspects, in particular in such a complex system as an intermodal network. These stochastic elements can be viewed generally as perturbations of the expected system behaviour which has been considered in defining the off-line plan. For this reason, the off-line plan, when applied on-line, can turn out to be no more feasible, or still feasible but no more effective. During the on-line phase the system is monitored and, if the off-line plan is no more valid, a replanning procedure is applied.

A sketch of the decision phases of off-line and on-line planning is provided in Fig. 1. The planning procedure is defined for a given time horizon (that can be for instance a week). Orders relative to this time horizon can arrive at different time instants (because customers issue their orders whenever they want) but within a fixed deadline. Orders arriving after this deadline are not taken into account when making the transportation planning. After the deadline, the off-line plan is realized (by running a suitable optimization procedure) and, as a result, the off-line plan is decided and fixed. If between the deadline and the end of the planning horizon a criticality happens, the on-line procedure must be applied.

### III. OFF-LINE PLANNING

The objective of the off-line planning is to find an optimal solution to the problem of satisfying transportation orders by using the railway services provided on the considered network. This problem takes into account a given cost function, some specific constraints and the decisions concern the assignment of boxes to a given path in the network, to a sequence of trains (hence, a sequence of railway terminals) and to specific wagons on trains. Specifically, the off-line planning procedure we have devised is composed of two phases:

- *preprocessing phase*, aiming at defining the feasible sequences of trains over the network for moving the boxes of a given order from the origin to the destination (considering topographical characteristics, time delivery requirements about orders and train timetables);
- *optimization phase*, consisting in the solution of a mathematical programming problem (starting from data provided by the preprocessing) such that each box is assigned

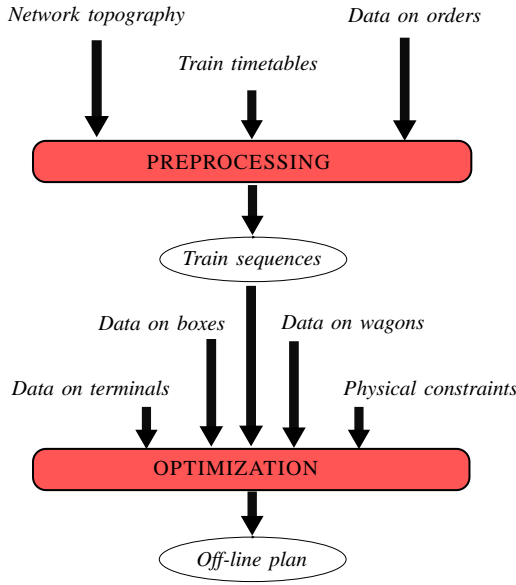


Fig. 2. The off-line planning procedure.

to a train sequence and to a wagon of each train composing the sequence in order to minimize all costs.

A sketch of the off-line planning procedure is provided in Fig. 2. In the preprocessing phase orders are processed one at a time because the objective is to determine all the feasible train sequences for that order (on the base of data about the network, about train timetables and about time specifications of the considered order). The output of the preprocessing phase, i.e. the feasible train sequences for each order and the relative costs, is then used as an input for the optimization procedure. This latter needs other input data, such as some information about each box of each order, some information on wagons of each train and other physical characteristics of the system. The optimization procedure, based on the solution of a mathematical programming problem, provides as an output the off-line plan (optimal assignment of boxes to train sequences and to wagons on trains). In the following subsections, the preprocessing phase and the optimization one will be described in detail.

### A. Preprocessing phase

The objective of the preprocessing phase is the definition of the sequences of trains that can be used for serving a given order; we refer to sequences of trains because each box is generally transported by more than one train within the path from its origin to its destination. This computation can be done on the base of data about the network topography and about static information on orders, such as their origins and destinations and the time delivery requests. The objective of this preprocessing computation is not the assignment of orders to trains, which of course must take into account the availability of resources and will be matter of the optimization problem; for this reason, the preprocessing can be realized considering orders one at a time.

The considered railway network is represented by means of an oriented graph  $\mathcal{G} = (\mathcal{N}, \mathcal{L})$ , where  $\mathcal{N}$  is the set of nodes (railway terminals) and  $\mathcal{L}$  is the set of links in the network.

The input data of the preprocessing phase are listed in the following:

- $on_o \in \mathcal{N}, dn_o \in \mathcal{N}, o = 1, \dots, O$ , i.e. the origin and destination nodes for order  $o$ ;
- $t_o^{in}, o = 1, \dots, O$ , i.e. the time instant in which goods of order  $o$  are ready at the origin node;
- $t_{min,o}^*, t_{max,o}^*, o = 1, \dots, O$ , i.e. the minimum delivery time instant and the maximum delivery time instant for order  $o$ ;
- $\alpha_o, \beta_o, o = 1, \dots, O$ , i.e. the cost term related to an early and a late delivery for order  $o$  (both are defined per box and per time unit);
- $\mathcal{L}_r \subseteq \mathcal{L}, r = 1, \dots, R$ , i.e. the path covered by train  $r$ , defined as a sequence of links;
- $dt_{r,l}, at_{r,l}, l \in \mathcal{L}_r, r = 1, \dots, R$ , i.e. the expected departure time at the origin of link  $l$  and the expected arrival time at the end of link  $l$  for train  $r$  (train timetable);
- $\delta_n, n \in \mathcal{N}$ , i.e. a cost term related to the handling operations of one box at terminal  $n$  (defined per box);
- $\rho_n, n \in \mathcal{N}$ , i.e. a cost term related to the storage of one box at terminal  $n$  (defined per box and per time unit);
- $\mathcal{R}_n, n \in \mathcal{N}$ , i.e. the set of trains including node  $n$  in their path.

When an order is considered in the preprocessing phase, three decision levels are faced. Firstly, the possible paths on the network are computed and this only depends on the network topography. Secondly, the trains available for serving the considered order are defined (this latter decision depends on delivery times for the order). Then, some cost terms are computed. These three phases of preprocessing are described in the following.

In order to compute the possible paths for a given order, it is necessary to know the origin and destination nodes for the order and the network topography. A number of alternative paths  $Np_o$  connecting the origin and the destination of order  $o$  is defined; each of these paths is described as a sequence of links, gathered in a set  $\mathcal{L}_{o,p} \subseteq \mathcal{L}, p = 1, \dots, Np_o$ . Note that this is a completely static information, which results to be the same for each order having the same origin-destination pair.

The definition of the set of feasible trains for serving order  $o$  is based on a specific iterative procedure (we do not report the complete algorithm in this paper for space limitations). The basic idea of this procedure is considering all train timetables and looking for the feasible trains for each path. Specifically, the proposed algorithm starts from the last link of each path, the set of trains traveling on this link and arriving at the end of it within the requested time window is defined and a corresponding time window referred to the origin of the last link is computed. Then, the previous link is considered and another set of feasible trains on that link is defined. This procedure is iterated until reaching the first link of the path, verifying that the first train is feasible with the time instant in

which goods are available at the origin railway terminal. The outputs of this procedure are the following:

- for each order  $o$  and for each path  $p$ ,  $p = 1, \dots, Np_o$ ,  $Ns_{o,p}$  train sequences are defined;
- each train sequence  $\mathbf{S}_{o,p,s}$ ,  $s = 1, \dots, Ns_{o,p}$ , is a vector  $1 \times |\mathcal{L}_{o,p}|$  listing the trains for each link of path  $p$  for order  $o$ ;
- $\mathcal{R}_{o,p,s}$ ,  $p = 1, \dots, Np_o$ ,  $s = 1, \dots, Ns_{o,p}$ , is the set of trains included in sequence  $s$  of path  $p$  for order  $o$ ;
- $\mathcal{S}_{o,p,n,r}^U$ ,  $p = 1, \dots, Np_o$ ,  $n \in \mathcal{N}$ ,  $r \in \mathcal{R}_n$  is the set gathering the indices of sequences of path  $p$  for order  $o$  such that boxes are unloaded from train  $r$  at node  $n$  (by definition, the elements of set  $\mathcal{S}_{o,p,n,r}^U$  must be  $\geq 1$  and  $\leq Ns_{o,p}$ );
- $\mathcal{S}_{o,p,n,r}^L$ ,  $p = 1, \dots, Np_o$ ,  $n \in \mathcal{N}$ ,  $r \in \mathcal{R}_n$  is analogous to the previous set but refers to loading operations;
- $\mathcal{S}_{o,p,n,r}^T$ ,  $p = 1, \dots, Np_o$ ,  $n \in \mathcal{N}$ ,  $r \in \mathcal{R}_n$  refers to transfer operations (i.e. trains that pass a given node and do not involve any loading/unloading operations).

Some cost terms are computed since they are associated with the sequence  $s$  of path  $p$  for order  $o$  and do not depend on planning decisions:

- $E_{o,p,s}$  is the earliness cost for each box: this cost term is computed starting from  $\alpha_o$  and considering the earliness determined by serving order  $o$  with sequence  $s$ ;
- $T_{o,p,s}$  is the tardiness cost for each box, computed starting from  $\beta_o$ , analogously to the earliness cost;
- $RT_{o,p,s}$  is the cost associated with railway terminals where boxes are loaded/ unloaded (it is computed as the sum of the cost terms  $\delta_n$  for each involved terminal);
- $ST_{o,p,s}$  is a storage cost related to the wait of boxes in all terminals involved in the considered sequence (computed considering  $\rho_n$  and the relative storage time length).

Note that all these cost terms depend not only on the sequence but also on the order, because they could be weighted in a different way for different orders, taking into account management and pricing aspects that are not considered here. The overall cost  $C_{o,p,s}$  for the sequence  $s$  of path  $p$  for order  $o$  is computed as follows:

$$C_{o,p,s} = \frac{1}{\zeta_{o,p}} (E_{o,p,s} + T_{o,p,s} + RT_{o,p,s} + ST_{o,p,s}) \quad (1)$$

where  $\zeta_{o,p}$  is a cost term relative to priority of paths. Actually, the planner could distinguish among the different paths connecting each origin-destination pair depending on a priority level, such that there is a primary path, then a secondary one, and so on. More specifically, we define  $\zeta_{o,p}$  as a parameter between 0 and 1, taking value 1 when path  $p$  for order  $o$  is the primary one and taking lower values for secondary, tertiary paths, and so on. In this way, being the sum of the previous defined cost terms divided by  $\zeta_{o,p}$ , this results to be a weighting term, which favors the choice of primary paths.

### B. Optimization phase

Once all the possible train sequences have been defined for each order, it is necessary to assign each box of any order

to one of such sequences. The assignment decision is taken by solving the optimization problem described in the present subsection. For stating and solving this problem, the output data of the preprocessing phase are used, as well as other data about quantities to be moved, physical constraints on trains and resource capacities, as listed in the following:

- $Nb_o$ ,  $o = 1, \dots, O$ , i.e. the number of boxes (either containers or swap bodies) associated with order  $o$ ;
- $\pi_{o,b}$ ,  $o = 1, \dots, O$ ,  $b = 1, \dots, Nb_o$ , i.e. the length of box  $b$  of order  $o$ ;
- $\omega_{o,b}$ ,  $o = 1, \dots, O$ ,  $b = 1, \dots, Nb_o$ , i.e. the weight of box  $b$  of order  $o$ ;
- $\Omega_r$ ,  $r = 1, \dots, R$ , i.e. the maximum weight that train  $r$  can move;
- $K_r$ ,  $r = 1, \dots, R$ , i.e. a cost term related to the use of train  $r$ ;
- $W_r$ ,  $r = 1, \dots, R$ , i.e. the number of wagons for train  $r$ ;
- $\Omega_{r,w}$ ,  $r = 1, \dots, R$ ,  $w = 1, \dots, W_r$ , i.e. the maximum bearable weight for wagon  $w$  of train  $r$ ;
- $\Pi_{r,w}$ ,  $r = 1, \dots, R$ ,  $w = 1, \dots, W_r$ , i.e. the length of wagon  $w$  of train  $r$ ;
- $\sigma_n$ ,  $n \in \mathcal{N}$ , i.e. the maximum number of handling operations (loading and unloading) for each train at terminal  $n$  (this term depends on the handling capacity provided by each terminal);
- $\mathcal{W}_{n,r}$ ,  $n \in \mathcal{N}$ ,  $r \in \mathcal{R}_n$ , i.e. the set of wagons of train  $r$  that can be worked in the terminal  $n$  (in fact, some physical constraints on railway terminals could cause a limitation in the number of train wagons where loading/unloading can be performed).

The planning problem is then stated as a linear binary integer programming (BIP) problem since the decision variables are required to be either 0 or 1. More specifically, the problem decision variables are:

- $y_{o,b,p,s} \in \{0, 1\}$ ,  $o = 1, \dots, O$ ,  $b = 1, \dots, Nb_o$ ,  $p = 1, \dots, Np_o$ ,  $s = 1, \dots, Ns_{o,p}$ , assuming value 1 if box  $b$  of order  $o$  is assigned to sequence  $s$  of path  $p$ , otherwise equal to 0;
- $x_{o,b,p,s,r,w} \in \{0, 1\}$ ,  $o = 1, \dots, O$ ,  $b = 1, \dots, Nb_o$ ,  $p = 1, \dots, Np_o$ ,  $s = 1, \dots, Ns_{o,p}$ ,  $r \in \mathcal{R}_{o,p,s}$ ,  $w = 1, \dots, W_r$ , assuming value 1 if box  $b$  of order  $o$  is assigned to wagon  $w$  of train  $r$  in sequence  $s$  of path  $p$ , otherwise equal to 0;
- variables  $z_r \in \{0, 1\}$ ,  $r = 1, \dots, R$ , assuming value 1 if train  $r$  is used, otherwise equal to 0.

The following problem can be written where the objective function to be minimized considers the cost terms associated with train sequences (computed in the preprocessing phase) and train costs.

*Problem 1:* Find

$$\min_{\substack{y_{o,b,p,s} \\ x_{o,b,p,s,r,w} \\ z_r}} \sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s=1}^{Ns_{o,p}} C_{o,p,s} \cdot y_{o,b,p,s} + \sum_{r=1}^R K_r \cdot z_r \quad (2)$$

subject to

$$\sum_{p=1}^{Np_o} \sum_{s=1}^{Ns_{o,p}} y_{o,b,p,s} = 1 \quad o = 1, \dots, O \quad b = 1, \dots, Nb_o \quad (3)$$

$$\sum_{w=1}^{W_r} x_{o,b,p,s,r,w} = y_{o,b,p,s} \quad o = 1, \dots, O \quad b = 1, \dots, Nb_o$$

$$p = 1, \dots, Np_o \quad s = 1, \dots, Ns_{o,p} \quad r \in \mathcal{R}_{o,p,s} \quad (4)$$

$$\sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s \in (S_{o,p,n,r}^L \cup S_{o,p,n,r}^T)} \sum_{w=1}^{W_r} \omega_{o,b} x_{o,b,p,s,r,w} \leq \Omega_r z_r$$

$$n \in \mathcal{N} \quad r \in \mathcal{R}_n \quad (5)$$

$$\sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s \in (S_{o,p,n,r}^L \cup S_{o,p,n,r}^T)} \pi_{o,b} x_{o,b,p,s,r,w} \leq \Pi_{r,w}$$

$$n \in \mathcal{N} \quad r \in \mathcal{R}_n \quad w = 1, \dots, W_r \quad (6)$$

$$\sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s \in (S_{o,p,n,r}^L \cup S_{o,p,n,r}^T)} \omega_{o,b} x_{o,b,p,s,r,w} \leq \Omega_{r,w}$$

$$n \in \mathcal{N} \quad r \in \mathcal{R}_n \quad w = 1, \dots, W_r \quad (7)$$

$$\sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s \in (S_{o,p,n,r}^L \cup S_{o,p,n,r}^U)} \sum_{w=1}^{W_r} x_{o,b,p,s,r,w} \leq \sigma_n$$

$$n \in \mathcal{N} \quad r \in \mathcal{R}_n \quad (8)$$

$$\sum_{o=1}^O \sum_{b=1}^{Nb_o} \sum_{p=1}^{Np_o} \sum_{s \in (S_{o,p,n,r}^L \cup S_{o,p,n,r}^U)} x_{o,b,p,s,r,w} = 0$$

$$n \in \mathcal{N} \quad r \in \mathcal{R}_n \quad w \notin \mathcal{W}_{n,r} \quad (9)$$

$$y_{o,b,p,s} \in \{0, 1\} \quad o = 1, \dots, O \quad b = 1, \dots, Nb_o$$

$$p = 1, \dots, Np_o \quad s = 1, \dots, Ns_{o,p} \quad (10)$$

$$x_{o,b,p,s,r,w} \in \{0, 1\} \quad o = 1, \dots, O \quad b = 1, \dots, Nb_o$$

$$p = 1, \dots, Np_o \quad s = 1, \dots, Ns_{o,p} \quad r \in \mathcal{R}_{o,p,s} \quad w = 1, \dots, W_r \quad (11)$$

$$z_r \in \{0, 1\} \quad r = 1, \dots, R \quad (12)$$

Constraints (3) impose that each box of each order is assigned to one and only one train sequence, while (4) impose the relation between  $y_{o,b,p,s}$  and  $x_{o,b,p,s,r,w}$  variables. Moreover, constraints (5) concern the maximum weight that each train can bear; these constraints define also the relation between  $x_{o,b,p,s,r,w}$  and  $z_r$  variables. Constraints (6) and (7) impose that boxes assigned to wagons are compatible with the wagon length and the weight limitations for each wagon.

Constraints (8) regard the maximum handling operations that can be performed for each train at a given terminal. Finally, constraints (9) impose that, if in a given terminal some wagons cannot be loaded/unloaded for any physical constraints, the corresponding decision variables are not assigned.

As already noted, Problem 1 is a linear BIP problem that can be solved by standard mathematical programming techniques and tools.

#### IV. ON-LINE PLANNING

The on-line planning aims at defining new assignments of orders to train sequences when a criticality happens during the system evolution. Different types of critical situations have been taken into account, such as:

- trains delayed or out of order;
- links delayed or out of order;
- nodes delayed or out of order.

All these criticalities can be seen as disturbances to the system dynamics if compared to the off-line planning. When the system faces a critical situation, the on-line planning is supposed to be realized in 3 phases:

- *analysis phase*, aiming at evaluating the impact of the present criticality; in case this impact is considered serious, the local optimization is applied;
- *local optimization phase*, with the function of finding a local (i.e. generally sub-optimal) solution that replaces the off-line plan by facing the present system perturbation;
- *global optimization phase*, aiming at replanning all orders by solving again Problem 1 (of course, taking into account the present criticality); this phase is run only when the local optimization has failed.

The general decision scheme of the on-line planning procedure is provided in Fig. 3. The detailed off-line plan, as well as all static data, must be known during the on-line system operations. Whenever a criticality happens, it must be considered and categorized (delay, out of order, and so on). Then, the analysis phase can be applied in order to verify the impact of the perturbation on the system: if there are no critical boxes, this means that the criticality is not influential; if, on the contrary, the perturbation is too serious, the situation cannot be faced by a replanning but only manually by human operators. In the intermediate case in which there are some critical boxes, the local optimization phase is applied. Two possible cases arise: if a local solution is found, it is applied (hence, changing the off-line plan); if the local optimization procedure is not able to find a feasible solution, the global optimization is called (thus solving a similar problem to the one faced off-line). Again, there are two possible cases: if a global solution exists, it is applied; otherwise, the situation must be treated manually.

##### A. Analysis

The analysis phase is applied whenever a criticality happens. It has two main functions:

- evaluating the effects of the present criticality on the off-line plan;

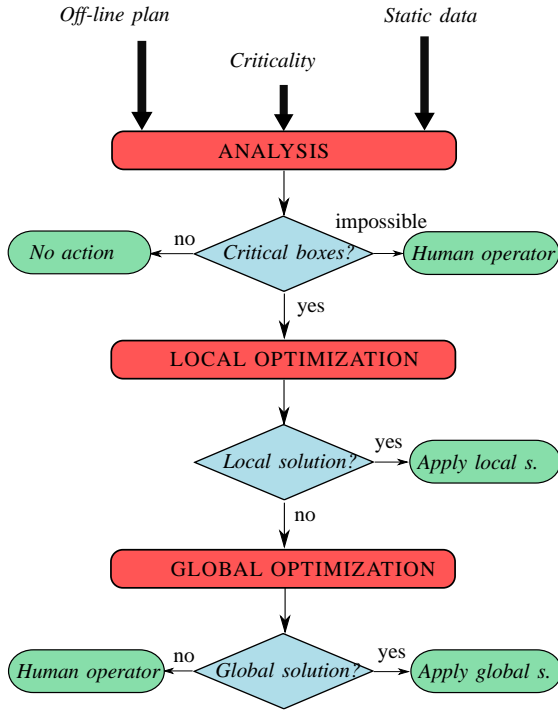


Fig. 3. The on-line planning procedure.

- on the basis of this evaluation, deciding whether a local replanning is necessary/possible or not.

The analysis phase aims at evaluating how the present criticality affects the system behaviour. For instance, it is possible that the train capacity is exceeded, some physical constraints are not met, one or more orders are delayed, and so on. Generally speaking, three possible decisions can be taken after the analysis phase:

- 1) the criticality is considered irrelevant, hence the off-line plan is still valid;
- 2) the criticality requires a local replanning;
- 3) the criticality is so serious that it cannot be solved by replanning but only manually by a human operator.

More specifically, the analysis phase is realized in two steps. Firstly, the set of boxes for which the assignment (to a train sequence, as in the off-line plan) is no more feasible is computed. This infeasibility can happen because, with the present criticality, these boxes are too late or because they are assigned to a train that is out of service or because their path includes a link/node that is out of service, and so on. In this way a set  $\tilde{\mathcal{O}}$  of *critical orders* and a set  $\tilde{\mathcal{B}}_o$  of *critical boxes* for each critical order are computed. If there are not critical boxes, i.e.  $\tilde{\mathcal{O}} = \emptyset$  and  $\tilde{\mathcal{B}}_o = \emptyset$ , we are in case 1) corresponding to an irrelevant criticality. For every box  $b \in \tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , a criticality index is computed (related to the delay and the number of critical boxes of the same order).

The elements of each set  $\tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , are analysed starting from the one characterized by the highest criticality index and going on until the one with the lowest criticality index. For every critical box  $b \in \tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , all train sequences associated

with the corresponding order,  $\mathbf{S}_{o,p,s}$ ,  $s = 1, \dots, N_{s_{o,p}}$ ,  $p = 1, \dots, N_{p_o}$ , are analysed and the infeasible train sequences are cancelled. This analysis depends on the type of criticality:

- link out of order: train sequences including that link or train sequences including a train that covers the link out of order (before passing in the link included in the train sequence) are cancelled;
- node out of order: train sequences including that node or train sequences including a train that includes the node out of order (before passing in the link included in the train sequence) are cancelled;
- train out of order: train sequences including the train out of order are cancelled;
- delays on nodes, links and trains: the evaluation is analogous to the one of out of order but, in this case, the verification is whether the boxes arrive at destination late or in time; only in the former case, the related train sequences are cancelled.

If at the end of this procedure there is at least one feasible train sequence for every box  $b \in \tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , we are in case 2), then the local optimization phase is applied. Otherwise, if there is at least one critical box such that no feasible train sequences have been found, we are in case 3), i.e. the criticality cannot be solved by local/global optimization and the intervention of a human operator is required.

### B. Local optimization

As already stated, the local optimization phase is applied if for every box  $b \in \tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , at least one feasible train sequence has been found. Starting from the first element of  $b \in \tilde{\mathcal{B}}_o$  (that is the most critical box) the following procedure is applied:

- all the train sequences are analysed for the considered box, in particular verifying whether the box can stay in those trains in the relative link (verifying the constraints on weight and length);
- if for a box a feasible train sequence is found, this box is cancelled from the set of critical boxes;
- if for a box a feasible train sequence is not found, it remains in the set of critical boxes and the following box is considered;
- if all boxes have been analysed and there are no more critical boxes, the procedure is terminated successfully (a local solution has been found);
- if there are still critical boxes, it is necessary to verify whether there is at least one feasible train sequence for one remaining box; in this latter case, the procedure is applied again, otherwise it ends as a failure, thus the global optimization is applied.

### C. Global optimization

The global optimization phase is applied when the local one has failed. This means that analysing all the critical boxes  $b \in \tilde{\mathcal{B}}_o$ ,  $o \in \tilde{\mathcal{O}}$ , and the relative train sequences, a feasible assignment of these boxes to these train sequences has not been found. Even though a local solution has not been found,

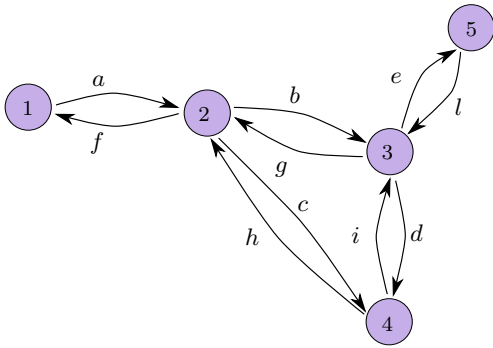


Fig. 4. An example network.

it is in generally possible to solve the present criticality by means of the global optimization phase.

The global optimization procedure corresponds to the solution of an optimization problem derived from Problem 1. In general, the problem to be solved on-line is given by Problem 1 in which some of the off-line decision variables are now fixed. This happens for instance for those boxes that have already been loaded on a train (hence, that assignment to that train is no more matter of decision). Otherwise, if the criticality happens after the time instant in which the off-line planning has been realized but before the beginning of the planning horizon, Problem 1 is run in its complete form.

## V. EXAMPLE

In this section, a simple example is reported in order to clarify the presented planning procedure, both in the off-line and in the on-line version. The results reported in the following have been found by using a planning tool implementing the proposed off-line and on-line procedure. All the modules have been coded in C# and Cplex 10.0 solver has been used for the solution of the mathematical programming problem (Problem 1). The effectiveness of the overall decisional procedure has been tested considering some case studies based on realistic data. In this section, we report a simple example in order to better clarify the concepts.

In fig. 4 the considered simple network is shown. Moreover, we suppose that the planning horizon corresponds to one single day (therefore, in the reported timetables and time instants the reference to the date is neglected and only the reference to hours and minutes is considered). The purpose of this section is to clarify the main ideas at the base of the planning procedures described above; for this reason, only the most significant data are considered, while some others are not indicated.

The example considers four trains ( $R = 4$ ) whose main data are reported in Table I. For instance, if we consider train  $r = 1$ , it covers links  $a$ ,  $b$  and  $d$  with the following timetable: departure from from node 1 at 2.00, arrival at node 2 at 7.00; departure from node 2 at 8.30, arrival at node 3 at 12.00; departure from node 3 at 15.00, arrival at node 4 at 17.30. The overall cost associated with the use of train  $r = 1$  is  $K_1 = 90$ .

TABLE I  
DATA ABOUT TRAINS.

	$\mathcal{L}_r$	$dt_{r,l}$	$at_{r,l}$	$K_r$
$r = 1$	$\{a, b, d\}$	2.00 8.30 15.00	7.00 12.00 17.30	90
$r = 2$	$\{a, c\}$	3.30 10.30	9.30 18.00	200
$r = 3$	$\{e\}$	14.00	20.00	170
$r = 4$	$\{a, b, e\}$	5.00 12.00 18.30	10.30 17.00 22.30	230

Each train is characterized by a given (small) number of wagons, i.e.  $W_1 = 3$ ,  $W_2 = 3$ ,  $W_3 = 4$  and  $W_4 = 2$ . The maximum weight and length for each wagon are reported in Table II. Considering again train  $r = 1$ , it is composed of 3 wagons: each wagon can bear a maximum weight of 65, 70, 70 tons, respectively, and has a length of 18, 22, 22 metres, respectively.

TABLE II  
DATA ABOUT WAGONS.

	$\Omega_{r,w}$ , $w = 1, \dots, W_r$	$\Pi_{r,w}$ , $w = 1, \dots, W_r$
$r = 1$	65 70 70	18 22 22
$r = 2$	70 70 80	22 22 25
$r = 3$	70 65 70 80	22 18 22 25
$r = 4$	80 80	25 25

We consider  $O = 2$  orders to be served, whose data are reported in Table III. For instance, order  $o = 1$  is a request of moving boxes from node 2 to node 4; goods are ready in node 2 at 6.00 and the delivery time window is between 18.00 and 20.00.

TABLE III  
DATA ABOUT ORDERS.

	$on_o$	$dn_o$	$t_o^{in}$	$t_{min,o}^*$	$t_{max,o}^*$
$o = 1$	2	4	6.00	18.00	20.00
$o = 2$	1	5	1.00	19.00	23.00

These orders correspond to a certain number of boxes, i.e.  $Nb_1 = 3$ , and  $Nb_2 = 2$ . The weight and length for each box are reported in Table IV. Considering again order  $o = 1$ , it is composed of 3 boxes, weighing 30, 20, 18 tons, respectively, and being 12.5, 8, 8 metres long, respectively.

TABLE IV  
DATA ABOUT BOXES.

	$\omega_{o,b}$ , $b = 1, \dots, Nb_o$	$\pi_{o,b}$ , $b = 1, \dots, Nb_o$
$o = 1$	30 20 18	12.5 8 8
$o = 2$	27 28	12.5 13.5

Consider now the preprocessing phase for order  $o = 1$ . Two paths ( $Np_1 = 2$ ) are defined, as follows:

$$\mathcal{L}_{1,1} = \{b, d\} \quad (13)$$

$$\mathcal{L}_{1,2} = \{c\} \quad (14)$$

Preprocessing (taking into account train timetables and order time requests) finds one train sequence for each path ( $Ns_{1,1} = 1$ ,  $Ns_{1,2} = 1$ ), as follows:

$$\mathbf{S}_{1,1,1} = [1 \ 1] \quad (15)$$

$$\mathbf{S}_{1,2,1} = [2] \quad (16)$$

This means that order  $o = 1$  can be served in two possible ways: train  $r = 1$  on links  $b$  and  $d$  (in this case, leaving from node 2 at 8.30 and arriving at node 4 at 17.30); train  $r = 2$  on link  $c$  (leaving from node 2 at 10.30 and arriving at node 4 at 18.00). The cost  $C_{o,p,s}$  associated with each train sequence is then computed:

$$C_{1,1,1} = 10 \quad (17)$$

$$C_{1,2,1} = 5 \quad (18)$$

Consider now the preprocessing computation for order  $o = 2$ . In this case, one path ( $Np_2 = 1$ ) is considered, that is:

$$\mathcal{L}_{2,1} = \{a, b, e\} \quad (19)$$

Two train sequences for this path are found ( $Ns_{2,1} = 2$ ):

$$\mathbf{S}_{2,1,1} = [1 \ 1 \ 3] \quad (20)$$

$$\mathbf{S}_{2,1,2} = [4 \ 4 \ 4] \quad (21)$$

Order  $o = 2$  can be then served in two ways: train  $r = 1$  on links  $a$  and  $b$  and train  $r = 3$  on link  $e$  (leaving from node 1 at 2.00 and arriving at node 5 at 20.00); train  $r = 4$  on links  $a$ ,  $b$  and  $e$  (leaving from node 1 at 5.00 and arriving at node 5 at 22.00). Note that the first sequence involves a train change at node 3. This is possible because the train connection is respected; in fact, train  $r = 1$  is expected to arrive at node 3 at 12.00 and the scheduled departure time of train  $r = 3$  from node 3 is 14.00.

The cost  $C_{o,p,s}$  associated with each train sequence is:

$$C_{2,1,1} = 5 \quad (22)$$

$$C_{2,1,2} = 20 \quad (23)$$

By applying the optimization procedure, hence by running Problem 1, the optimal values of the decision variables are found. The variables  $y_{o,b,p,s}$  assuming value 1 in the optimal solution are the following:

$$y_{1,1,1,1} = 1 \quad y_{1,2,1,1} = 1 \quad y_{1,3,1,1} = 1 \quad (24)$$

$$y_{2,1,1,1} = 1 \quad y_{2,2,1,1} = 1 \quad (25)$$

This means that all boxes of order  $o = 1$  are assigned to the sequence  $\mathbf{S}_{1,1,1} = [1 \ 1]$ , whereas all boxes of order  $o = 2$  are assigned to the sequence  $\mathbf{S}_{2,1,1} = [1 \ 1 \ 3]$ . Moreover, the variables  $x_{o,b,p,s,r,w}$  whose optimal value is equal to 1 are:

$$x_{1,1,1,1,1,1} = 1 \quad x_{1,2,1,1,1,2} = 1 \quad x_{1,3,1,1,1,3} = 1 \quad (26)$$

$$x_{2,1,1,1,1,2} = 1 \quad x_{2,2,1,1,1,3} = 1 \quad (27)$$

$$x_{2,1,1,1,3,1} = 1 \quad x_{2,2,1,1,3,2} = 1 \quad (28)$$

The first box of order  $o = 1$  is assigned to wagon  $w = 1$  of train  $r = 1$ ; the second box of order  $o = 1$  is assigned to wagon  $w = 2$  of train  $r = 1$ ; the third box of  $o = 1$  is assigned to the third wagon of train  $r = 1$ . The two boxes of order  $o = 2$  are assigned to the second and third wagon of train  $r = 1$ , respectively, and to the first and second wagon of train  $r = 3$ . Note that the two orders share train  $r = 1$  in link  $b$ : all boxes can be transported in this same train on this link because weight and length constraints are met.

Obviously, variables  $z_r$ , taking value 1, are  $z_1 = 1$ ,  $z_3 = 1$  since only trains  $r = 1$  and  $r = 3$  are assigned to at least one box.

Suppose now that, when on-line applying this plan, train  $r = 3$  is completely out of order and cannot leave (of course we suppose to know about this problem when boxes are still at their origin destination and have not yet left). Applying the analysis phase, boxes of order  $o = 2$ , which have been assigned to a sequence including train  $r = 3$ , are critical. Hence,  $\tilde{\mathcal{O}} = \{2\}$  and  $\tilde{\mathcal{B}}_1 = \{1, 2\}$ . For each box  $b \in \tilde{\mathcal{B}}_1$  a feasible train sequence exist, i.e.  $\mathbf{S}_{2,1,2} = [4 \ 4 \ 4]$ . Therefore, the local optimization phase is applied.

The local optimization phase corresponds to analyse whether it is possible to assign the critical boxes to this train sequence or not. In this case, train  $r = 4$  is empty, thus all boxes of order  $o = 2$  can be assigned to it. This is the result of the local optimization and this replanning action is applied to the system.

If, for instance, the criticality to face were link  $b$  out of order, the critical boxes would be again  $\tilde{\mathcal{O}} = \{2\}$  and  $\tilde{\mathcal{B}}_1 = \{1, 2\}$ , i.e. all boxes of order  $o = 2$ . Nevertheless, in this case no feasible train sequences can be found for these boxes because there is only one path for order  $o = 2$  and it includes link  $b$ .

## VI. CONCLUSIONS

In this paper we have described an off-line planning procedure and an on-line control framework for a railway network. The objective is to serve different transportation requests coming from different customers, with different origin-destination pairs and different characteristics. The off-line planning approach is composed of two phases, a preprocessing analysis considering one order at a time and the solution of linear BIP problem. The on-line procedure, aiming at facing unforeseen system perturbations, is composed of three phases: analysis, local optimization and global optimization.

All the modules have been implemented in C#. The mathematical programming problem has been solved by using Cplex 10.0 solver. The effectiveness of the overall decisional procedure has been tested considering some case studies based on realistic data. The most challenging phase with regard to the computational times is certainly the off-line planning phase in which all problem data are considered. Of course, the computational times depend on the dimensions of the problem, thus the number of orders, the number of boxes per order, the number of paths and of sequences for each path, the number of wagons for each train. In any case, the time required by Cplex

to find the optimal solution is generally acceptable since the problem structure is rather simple; this happens because the combinatorial aspects (such as the search of path and trains in the network) of the overall decision problem are treated in the preprocessing phase. An extensive evaluation of the computational complexity of the proposed scheme is matter of present and future research.

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